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# Light propagation and optical filtering properties of one dimensional Pascal plasma photonic multilayers

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# ABSTRACT

We numerically study the propagation of electromagnetic waves in Pascal plasma photonic crystals. For this purpose, the transfer matrix method is used. This method is based on defining multiple matrices for moving waves in interfaces and layers. Other methods such as finite difference time range can also be used, but are more suitable for higher-dimensional photonic crystals. We investigate the effect of dielectric layer refractive index, plasma electron density, total system length, wavelength angle, and the number of photon crystal layers on the transfer coefficient. A pseudo-code to create the Pascal multilayer is also presented. Finally, we describe the properties of the photonic bandgap, including position and width, because the parameters mentioned are different. We showed that as the refractive index  $n_1$  increases, the position of the gaps alternates redshifted and blueshifted (ie, decreasing and increasing the frequency and energy of the photon). Also, with increasing the refractive index  $n_1$ , the width of the gaps decreased and increased. As the collision angle  $\theta_0$  grew, the transmission edge blueshifted.

## 1. Introduction

Photonic crystals are usually defined as artificial structures possessing periodic or quasi-periodic refractive indices [1]. The incident light scatters at the interfaces of the photonic crystals. The superposition of scattering waves can produce photonic bandgaps that are analogous to the electronic bandgap structure in a semiconductor system. Since the first discovery of photonic crystals in 1987 [2-3]. they have found wide applications in optical devices, including optical fibers [4], absorbers [5], filters [6], and sensors [7]. In photonic bandgaps, the electromagnetic wave propagation is prohibited due to the Bragg scattering interference concept. So far, the researchers have used different dispersive or dissipative media to build a tunable photonic crystal, including semiconductors [8], metals [9], and plasma [10], superconductors [11]. In 2004, the terminology of plasma photonic crystals was first introduced by Hojo and coworkers [12]. According to them, the plasma photonic crystals is an efficient way of employing the plasma properties in microwave range [13]. Thereafter, they obtained potential utilizations in different devices, including the filter [14], omnidirectional reflector [15], and optical switching [16].

To produce the photonic multilayer, different arrangements of the alternative layers such as constant

total length photonic crystals [17] have so far been investigated. However, quasi-periodic structures with layer placed based on a recursion relation have crucial importance because they can exhibit different exciting properties. The most well-known sequences are Fibonacci [18], Cantor [19], Thue-Mores sequence [20], and the respectively. Besides, the optical properties of graphene mono- and multilayers [21], rectangular to semi-sinusoidal layers [22], three-component photonic crystals [23], nonlinear photonic crystals [24], photonic crystals comprising three phases nanocomposite layer [25], Photonic Crystals with a Staggered Structure [26], ternary plasma photonic crystals [27], Plasma Photonic Crystals Having Exponentially Graded Materials [28], disordered photonic crystals [29], double graded hyperbolic, exponential and linear index materials embedded onedimensional photonic crystals [30], cylindrical magnetized plasma photonic crystals [31], photonic crystals containing graphene-based hyperbolic metamaterials [32], photonic crystals of porous silicon [33], photonic crystals including MoS2 monolayer [34], photonic crystals composed of plasma and mu-negative materials [35], hybrid multifunctional YaBa2Cu307 photonic crystals [36], etc.

In the current paper, we have described the optical transmission through one-dimensional Pascal plasma photonic crystals utilizing the transfer matrix method. Up to our knowledge this structure has not been so far

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studied. The transfer matrix method is based on defining some matrices for traveling waves in interfaces and the layers. Other methods such as finite difference time domain can also be used but it is more appropriate for higher-dimensional photonic crystals. However, for one dimensional photonic crystal, the transfer matrix method is more efficient and faster. This paper is organized as follows. In section 2, we have presented the mathematical backbone of the present papers in addition to the algorithm to produce the Pascal photonic crystal refractive index profile. In section 3, we have described the obtained figures. Finally, in the conclusion section we have present some concluding sentences.

## 2. Formalism

The periodic or quasi-periodic nature of refractive index changes for a typical photonic crystal leads to the photonic bandgaps. Here, we create the photonic crystal with a refractive index profile according to the Pascal triangle arrangement. In figure 1, we have presented the variation of the refractive index n as a function of the position x. each layer is devoted to a material type (e.g. plasma or dielectric). Layers are alternatively composed of plasma with electron density  $n_e$  and dielectric material with refractive index  $n_1$ . Two configurations of the Pascal plasma photonic crystals are presented in panels (A) and (B).

The entry of each row and column in Pascal's triangle obeys this formula  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ . Where n and k indicate n<sup>th</sup> row and k<sup>th</sup> column of Pascal's triangle respectively. That is, to produce all columns of one row, all possible combinations of n should be generated. Let us denote all entries of one row by  $A_n$ . Here, we have used  $A_n$  to construct two potentials  $v_1$  and  $v_2$ . The width of each layer  $v_1$  is directly proportional to each entry of  $A_n$ , while layers' width  $v_2$  is inversely proportional to entries of  $A_n$ . The amplitude of  $v_1$  and  $v_2$  is set to zero, except for their odd layers where the amplitude is one.

Table 1 shows Algorithm to produce the Pascal photonic crystal refractive index profile. A one-dimensional Pascal plasma photonic crystal is assumed. The plasma layer is created from a solid-state plasma. We utilize the plasma and dielectric refractive indices as n<sub>1</sub> and n<sub>p</sub>, respectively. Besides, by assuming a non-magnetized cold plasma, we can use  $n_p = (1 - w_p^2 / w^2)^{1/2}$ . Where,  $w_p = (n_e e^2 / m e_0)^{1/2}$ 

m,  $n_{e,}$  and e is the plasma frequency, the electron mass, electronic plasma density, and electron charge, respectively [37].

 Table 1

 Algorithm to produce the Pascal photonic crystal refractive index profile

Algorithm  $v_1$  and  $v_2$ 

Input: n Output: Matrix $v_1$ and $v_2$			
 1. For k = 0 To n Step 1			
2.	$A_n(\mathbf{k}) = \mathbf{n}$ choose k		
3. For	3. For k = 0 To n Step 1		
4.	If k corresponds to odd layer		
5.	Add $A_n(\mathbf{k})$ ones to $v_1$		
6.	Add $SUM(A_n(k))/A_n(k)$ ones to $v_2$		
7.	Else		
8.	Add $A_n(\mathbf{k})$ zeros to $v_1$		
9.	Add $SUM(A_n(k))/A_n(k)$ zeros to $v_2$		

For the TE (TM) electromagnetic wave, the electric field (magnetic field) is assumed to be perpendicular to the planes of the photonic crystals. The electric and magnetic fields at two positions in the adjacent layers are related by [38],

$$M\left[\Delta z\right] = \begin{bmatrix} \cos\left(k_{z}^{j}\Delta z\right) & \frac{-i\sin\left(k_{z}^{j}\Delta z\right)}{p_{i}}\\ -ip_{i}\sin\left(k_{z}^{j}\Delta z\right) & \cos\left(k_{z}^{j}\Delta z\right) \end{bmatrix}$$
(1)

Where  $k_z^{\ j} = (\omega/c) n_j \cos(\theta_j)$  is the z-component of the wave vector in the layer. Also,  $p_j = \sqrt{\varepsilon_j / \mu_j} \cos(\theta_j)$ , where  $\cos \theta_j = \sqrt{1 - (n_0^2 \sin^2 \theta_0 / n_j^2)}$ . Moreover, c and  $\theta_j$  are the speed of light in vacuum and the beam angle in layer j.  $n_0$  is also the refractive index of the free space and  $n_j = \sqrt{\varepsilon_j \mu_j}$  denotes the k<sup>th</sup> layer refractive index [39], where,  $\varepsilon_i$  and  $\mu_i$  are the permittivity and the permeability of the k<sup>th</sup> layer materials, respectively. Then, the total transfer matrix due to the N-layers reads,

$$M^{Tot} = \prod_{j=1}^{N} M_{j} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$
(2)

The dimensionless transmission coefficient can now be written as,

$$t = \frac{2p_0}{\left(M_{11} + M_{12}p_s\right)p_0 + \left(M_{21} + M_{22}p_s\right)}$$
(3)

where  $p_0 = n_0 cos(\theta_0)$  and  $p_s = n_s cos(\theta_s)$ , where n<sub>0</sub>, n<sub>s</sub> are the environment refractive indices before the first layer and after the last layer, respectively. Also,  $\theta_0$  and  $\theta_s$  are the incident angle and the angle at which the light wave departs from the photonic crystal, respectively. The TM waves are

(4)

also described by using  $p_j = \sqrt{\mu_j / \varepsilon_j} \cos(\theta_j)$ ,  $p_0 = \cos(\theta_0) / n_0$ and  $p_s = \cos(\theta_s) / n_s$  [40]. The associated transmittance T of the multilayer is obtained using



**Fig. 1.** Variation of the refractive index n as a function of the position x. each layer is devoted to a material kind. Layers alternatively composed from plasma with electron density  $n_e$  and dielectric material with refractive index  $n_1$ . P and D indicate the plasma and dielectric layers, respectively. Two configurations of the Pascal plasma photonic crystals are presented in panels (A) and (B)

#### 3. Results and Discussion

 $T = \frac{p_s}{p_0} \left| t \right|^2$ 

In the present study, we investigated the optical transmission properties of an  $\omega$ -frequency electromagnetic wave through some of the plasma Pascal photonic crystals. In each figure, the effect of seven values of a usual parameter is considered, unless otherwise stated. From Fig. 2 onwards, note that all transmission coefficients are between zero and one. It is only for presentation and comparison that the various modes are stacked

Figure 2 presents the variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. The effect of seven different values on the number of layers (NOL) is compared. We also assumed  $\theta$ 0=0, ne=2E19, n1=3, L=60mm. Fig.3 is also the same as Fig. 2, but for some type 2 plasma photonic crystals. When a traveling electromagnetic wave enters a multilaver system, some part of it reflects and some portion transmits. The transmitted wave from layer I can reflect from layer i+1. Then these waves can interfere. According to this interference concept, we can describe the electromagnetic wave transmission through a thick multilayer. Because the constructive and deconstructive interference can lead to passband and bandgaps in the diagram of the transmission coefficient as a function of the incident wave frequency  $\omega$ . When the transmissions coefficient reaches 1, cent percent transmission occurs. However, if the transmissions coefficient vanishes, then the propagation of the wave with that e=frequency

becomes forbidden. As Figs. 2 and 3 show, increasing the number of layers (NOL) leads to changing the position of the bandgap and passbands because changing the arrangement and thickness of the layers leads to different interference situations. In both Figs. 2 and 3, the initial transmission edge for approximately all systems is about 40 GHz. Although a slight deviation from this frequency is visible. In the following, we restrict ourselves to the interval [40 to 60] GHz for the type 1 structure.



Fig. 2. Variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of seven different values of the number of layers (NOL) are compared. We also assumed  $\theta_0$ =0,  $n_e$ =2E19,  $n_1$ =3, L=60mm



Fig. 3. The same as figure 2, but for some type 2 plasma photonic crystals

In Fig. 4, we have depicted the variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. The effect of seven different values of the system total lengths L are compared. We also assumed  $\theta_0=0$ ,  $n_e=2E19$ ,  $n_1=3$ , L=60mm, and NOL=10. As one can see, by changing L, different gap widths are created. Thus, L can be a good tuning tool. In the figure, we have illustrated the gap position shifts by some arrows when L changes. As these arrows show, by increasing L, some gaps' position blueshift (i.e., move toward higher frequencies), while the other redshift. Therefore, using L increment we can regulate the gap position too. Another fact is that, by increasing L, roughly speaking, the number

of peaks in the diagram of the transmission versus wave frequency increases.

Figure 5 shows the variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. The effect of seven different values of the dielectric refractive index n<sub>1</sub> is compared. We also assumed  $\theta_0=0$ , n<sub>e</sub>=2E19, L=60mm, and NOL=10. In this figure, as the refractive index n<sub>1</sub> increases, the gaps' position alternatively redshift and blueshift. We have illustrated this fact using the dashed and dotted arrows. As one may see, the arrows are alternatively placed. Also, there are some flat transmission passbands in the range of 50 to 60 GHz for some systems. Finally, as the refractive index n<sub>1</sub> increases, the widths of the gaps alternatively decrease and increase.



**Fig. 4.** Variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of seven different values of the system total lengths L is compared. We also assumed  $\theta_0=0$ ,  $n_e=2E19$ ,  $n_1=3$ , L=60mm, and NOL=10



Fig. 5. Variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of seven different values of the dielectric refractive index  $n_1$  is compared. We also assumed  $\theta_0{=}0,~n_e{=}2E19,~L{=}60mm,~and~NOL{=}10$ 

In Fig. 6, we have presented the variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of seven different values of the plasma layer electron densities  $n_e$  is compared. We also assumed  $\theta_0{=}0, n_1{=}3, L{=}60\text{mm}, \text{and NOL}{=}10$ . It is seen that, for small values of the plasma electron density  $n_e$ , the two marked gapes blueshift. However, for high values of ne,

the left gap blueshift, while the other redshift. Another fact is that, by increasing  $n_e$ , the gap widths increase and become pore flat (i.e., more perfect gaps). Finally, figure 7 presents the variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. The effect of three different values of the angle  $\theta_0$  is compared. We also assumed  $n_e=2E19$ ,  $n_1=3$ , L=60mm, and NOL=10. As this figure shows, when  $\theta_0$  increases, the transmission edge blueshift.



**Fig. 6.** Variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of seven different values of the plasma layer electron densities  $n_e~(\mathrm{cm}^{-3})$  are compared. We also assumed  $\theta_0\text{=}0$ ,  $n_1\text{=}3, L\text{=}60\text{mm}$ , and NOL=10



Fig. 7. Variation of the transmission coefficient T as a function of the incident electromagnetic wave frequency  $\omega$  for some type 1 plasma photonic crystals. Effect of three different values of the angle  $\theta_0$  are compared. We also assumed  $n_e$ =2E19,  $n_1$ =3, L=60mm, and NOL=10

#### 4. Conclusions

In this work, we investigated the transfer properties of single-dimensional Pascal plasma photon crystals. It is shown that the initial transmission edge for our proposed structures was about 40 GHz. As L increases, some gaps' position is blueshifted, while others redshifted and it seemed they do not follow an order. But, as the refractive index  $n_1$  increased, the gaps' position alternatively redshifted and blueshifted. Also, as the refractive index  $n_1$  increased, the gaps alternatively decreased and increased. When  $\theta_0$  increased, the transmission edge blueshifted.

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