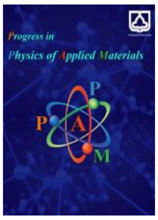
Progress in Physics of Applied Materials 7 (2027), Serial Number 14, 17-25

Semnan University

Progress in Physics of Applied Materials

journal homepage: https://ppam.semnan.ac.ir/



Quantum Teleportation Using Entangled Electron Spins in s- and d-Wave Superconductors

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ARTICLE INFO

Article history:
Received: 30 May 2026
Revised: 13 June 2026
Accepted: 17 June 2026
Published online: 30 June 2026

Keywords:
quantum teleportation;
d-wave superconductors;
quantum fidelity.

ABSTRACT

Quantum teleportation is one of the most remarkable protocols in quantum information science. It enables the transfer of an unknown quantum state from one location to another using quantum entanglement and classical communication. In solid state systems, superconductors provide a natural platform for generating entangled electron pairs through Cooper pairing. In this work, quantum teleportation is analyzed using entangled electron spins extracted from superconductors. The influence of the superconducting pairing symmetry on teleportation performance is examined. In particular, s-wave and d-wave superconductors are compared regarding their performance in the teleportation protocol, and the teleportation fidelity achievable using the spin entanglement of the two electron spins forming Cooper pairs is evaluated. For the d-wave case, a low-temperature approximation for the gap function is employed, where the gap function is proportional to the angle of the momentum vector with respect to the gap axis. The relationship between the teleportation fidelity and the distance between the two electron spins forming Cooper pairs, as well as the gap function, is also examined. Moreover, the d-wave superconductor exhibits distinctive features in relation to the fidelity. The distances at which the fidelity reaches the classical threshold are also determined. The analysis shows that superconductors can serve as a realistic resource for solid state quantum teleportation.

1. Introduction

Quantum entanglement is a fundamental resource for many protocols in quantum information processing, including quantum cryptography, quantum computation, and quantum teleportation [1-11]. Since the first proposal of quantum teleportation [7], the ability to transmit an unknown quantum state using shared entanglement and classical communication has been widely investigated in both theoretical and experimental contexts.

In condensed matter physics, superconductors represent an interesting natural source of entangled particles. According to the Bardeen–Cooper–Schrieffer (BCS) theory, electrons near the Fermi surface form bound pairs known as Cooper pairs [12]; moreover, gap fluctuations were discussed in this framework, and their role is also considered here in the context of quantum teleportation [13]. These pairs consist of two electrons with opposite momenta and opposite spins, forming a spin singlet state. Such a state is intrinsically entangled and therefore provides a potential resource for quantum information applications.

The possibility of extracting spin-entangled electrons from superconductors has attracted considerable interest [14-16]. When a Cooper pair is split into two spatially separated leads, the two electrons remain entangled. This phenomenon forms the basis for solid-state entanglement generation and can be used to implement quantum communication protocols.

While the generation and characterization of spin entanglement in superconductors have been investigated in previous studies, much less attention has been paid to using such entanglement as a quantum teleportation resource. In particular, earlier works mainly focused on

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Cite this article as:

Afzali R., 2027. Quantum teleportation using entangled electron spins in s- and d- wave superconductors. *Progress in Physics of Applied Materials*, *7*(1), pp.17-25. DOI: 10.22075/ppam.2026.41410.1235

the two-electron spin density matrix using Green's functions and on the quantification of entanglement in Cooper pairs [14,16]. In contrast, the present work uses the superconducting spin state as the quantum channel of a teleportation protocol and evaluates the corresponding teleportation fidelity. In the present work, spin-entangled electrons originating from superconducting Cooper pairs are used as the quantum channel of a teleportation protocol. The analysis is carried out using the superconducting two-electron density matrix and the Green-function formalism, which connect the teleportation fidelity to the microscopic superconducting correlations. The main d-wave case considered here is the $d_{x^2-y^2}$ symmetry. For this symmetry, a different set of gap values is considered, so additional numerical calculations are needed to evaluate the teleportation fidelity. Thus, the main new aspect of this work is the use of superconducting spin correlations as a teleportation channel, together with the numerical evaluation of the fidelity for different gap choices and critical. Also, the critical distance between the two electron spins forming Cooper pairs, at which classical teleportation begins, is calculated numerically and compared for the s-wave and d-wave case.

First, the quantum teleportation protocol for an unknown state via the Werner state of electron spins in Cooper pairs, which leads to a mixed final state, is discussed. Then, based on the Green's function approach, the two-electron density matrix associated with electrons extracted from the superconductor is presented. Using this density matrix, the teleportation fidelity achievable with this entangled resource is determined. This approach allows a direct connection to be established between superconducting correlations and quantum information performance, providing insight into the feasibility of superconductors as entanglement sources for quantum teleportation.

The present paper is organized as follows. In the "Theoretical Frame" section, the quantum teleportation protocol is first presented. Then, the quantum channel under consideration, namely; the Werner state associated with superconductors, is introduced. Finally, the average teleportation fidelity is calculated. In the "Results" section, the relevant plots are discussed. In the "Conclusions" section, the behavior of the curves and the main results are presented.

2. Theoretical Frame

2.1. Teleportation Protocol

An unknown qubit state specified by the density matrix ρ_{in} is considered for quantum teleportation. As the entangled resource, we employ a two-qubit Werner state [6] with parameter p, shared between Alice and Bob. Alice performs a Bell-state measurement on the input qubit together with her qubit from the Werner pair. After receiving the two classical bits from Alice, Bob applies the corresponding Pauli unitary operation to his qubit. As a result, Bob obtains the output state specified by the density matrix ρ_{out}, whose fidelity with the input state depends on the Werner-state parameter p. In the ideal case $p = 1$, the shared state becomes maximally entangled and the teleportation is perfect, yielding $\rho_{\text{out}} = \rho_{\text{in}}$. However, for the Werner state considered here, which is associated with a Cooper pair in a d-wave superconductor, the parameter p is not simply a constant parameter, but also depends on the relative distance between the two electrons forming the Cooper pair. This appears to be the first study in which a superconducting Werner state is employed as the entangled resource for quantum teleportation, with the additional feature that the entanglement parameter exhibits an explicit spatial dependence.

Alice wishes to teleport the following unknown single-qubit quantum state to Bob, i.e.,

$$\rho_{in} = |\psi\rangle\langle\psi| \tag{1}$$

In the standard quantum teleportation protocol, the unknown state to be teleported by Alice is typically defined as a single-qubit pure state, i.e.,

$$|\psi_{in}\rangle = \alpha|0\rangle+\beta|1\rangle \tag{2}$$

where α and β are complex probability amplitudes satisfying the normalization condition

$$|\alpha|^2 + |\beta|^2 = 1 \tag{3}$$

To integrate this pure state into our generalized density matrix formalism, we construct Alice's input density matrix as $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$. This explicit calculation yields:

$$\rho_{in} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} \tag{4}$$

Alice and Bob share an entangled pair of qubits (qubit 2 is with Alice and qubit 3 is with Bob) prepared in a Werner state. A Werner state is a statistical mixture of a maximally entangled state (a Bell state) and a completely mixed state (white noise), parameterized by $p \in [0,1]$ [6,14,16]

$$\rho_W = p|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4} I_2 \otimes I_2 \tag{5}$$

where $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$is the standard Bell state, and I_2 is the 2 × 2identity matrix.

It should be noted that one can express the Werner state in the Bell basis. One can define the four Bell states using the Pauli operators $\sigma_i \in \{I, X, Y, Z\}$as follows

$$|\beta_0\rangle = |\Phi^+\rangle = (I \otimes I)|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{6}$$

$$|\beta_1\rangle = |\Psi^+\rangle = (I \otimes X)|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{7}$$

$$|\beta_2\rangle = |\Psi^-\rangle = (I \otimes iY)|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{8}$$

$$|\beta_3\rangle = |\Phi^-\rangle = (I \otimes Z)|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \tag{9}$$

The identity matrix in the two-qubit space can be written as the sum of projection operators onto the Bell basis:

$$I_2 \otimes I_2 = \sum_{i=0}^{3} |\beta_i\rangle\langle\beta_i| \tag{10}$$

Substituting this relation into the definition of ρ_W, one obtains

$$\rho_W = p|\beta_0\rangle\langle\beta_0| + \tfrac{1-p}{4}\textstyle\sum_{i=0}^{3} |\beta_i\rangle\langle\beta_i| \tag{11}$$

The total initial state of the system is

$$\rho_{tot} = \rho_{in}^{(1)} \otimes \rho_W^{(2,3)} \tag{12}$$

Alice performs a Bell-state measurement on qubits 1 and 2. Her measurement operators are $P_m = |\beta_m\rangle\langle\beta_m|$ (for $m \in \{0,1,2,3\}$). According to a well-established mathematical identity in quantum information (the teleportation identity), the effect of the projection onto a Bell state on the tensor product of an input state and a Bell-basis state is

$$\langle\beta_m|_{1,2}\left(|\psi\rangle_1 \otimes |\beta_i\rangle_{2,3}\right) = \frac{1}{2}(\sigma_i\sigma_m)|\psi\rangle_3 \tag{13}$$

The probability of Alice obtaining outcome m is $\frac{1}{4}$. After measuring and obtaining outcome m, she sends this result to Bob via a classical channel. Bob then applies the corresponding unitary correction operator σ_m to his qubit. Bob's state, after applying the correction operator for a specific basis component i, becomes

$$\begin{aligned}\sigma_m(\sigma_i\sigma_m|\psi\rangle\langle\psi|\sigma_m\sigma_i^\dagger)\sigma_m &= (\sigma_m\sigma_i\sigma_m)\rho_{in}(\sigma_m\sigma_i\sigma_m)\\ &= (\pm\sigma_m^2\sigma_i)\rho_{in}(\pm\sigma_i\sigma_m^2)\\ &= \sigma_i\rho_{in}\sigma_i\end{aligned} \tag{14}$$

where the properties $\sigma_m\sigma_i = \pm\sigma_i\sigma_m$ and $\sigma_m^2 = I$ are used. Thus, the output state is independent of Alice's measurement outcome m as

$$\rho_{out} = \sum_{i=0}^{3} \lambda_i\, \sigma_i\rho_{in}\sigma_i \tag{15}$$

Now by substituting the values of λ_i into the output equation (with $\sigma_0 = I$), the following equation obtains

$$\begin{aligned}\rho_{out} = \frac{1+3p}{4} I_2\rho_{in}I_2\\ + \frac{1-p}{4}(X\rho_{in}X + Y\rho_{in}Y + Z\rho_{in}Z)\end{aligned} \tag{16}$$

To simplify the expression inside the parentheses, we use the well-known Pauli operator identity for single-qubit density matrices. Any single-qubit density matrix can be written as $\rho = \left(\frac{1}{2}\right)(I + \vec{r}\cdot\vec{\sigma})$. Therefore, one can write

$$X\rho_{in}X + Y\rho_{in}Y + Z\rho_{in}Z = 2I - \rho_{in} \tag{17}$$

Substituting this identity into Eq. (16) yields

$$\rho_{out} = p\rho_{in} + \frac{1-p}{2}I \tag{18}$$

This equation is the exact definition of a Depolarizing Channel. Mathematically, this means that teleportation via a Werner state is completely equivalent to passing the initial state through a depolarizing channel with parameter p. The output state is a mixed single-qubit state. To see this, we must check the purity, which is defined as $\mathrm{Tr}(\rho^2)$ where Tr is trace of a matrix [1].

The final state of teleportation using a Werner channel is:

$$\begin{aligned}Tr(\rho_{\mathrm{out}}^2) &= Tr\left(\left(p\rho_{\mathrm{in}} + (1-p)\frac{I}{2}\right)^2\right)\\ &= p^2 Tr(\rho_{\mathrm{in}}^2) + 2\cdot p(1-p)Tr(\rho_{\mathrm{in}})\frac{1}{2}\\ &+(1-p)^2\frac{Tr(I^2)}{4}\end{aligned} \tag{19}$$

Because of $\rho_{in}^2 = (|\psi\rangle\langle\psi|)(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi| = \rho_{in}$, $\mathrm{Tr}(\rho_{in}^2) = \mathrm{Tr}(\rho_{\mathrm{in}}) = 1$ and $\mathrm{Tr}(I^2) = \mathrm{Tr}(I) = 2$, one can write

$$\begin{aligned}Tr(\rho_{\mathrm{out}}^2) &= p^2 + 2\cdot p(1-p)\frac{1}{2} + (1-p)^2\frac{2}{4}\\ &= \frac{1+p^2}{2}\end{aligned} \tag{20}$$

Since $Tr(\rho_{\mathrm{out}}^2)$ is less than 1 for any $p < 1$, the output is a mixed state. Now, in the following section we proceed to identify the parameter p .

2.2. Teleportation Channel

Now, teleportation channel is investigated, that is the Werner state. Teleportation channel is done via the entangled electron spins of a Cooper pair of s- or d-wave superconductors. The foundation of superconductivity in many materials is described by the Bardeen–Cooper–Schrieffer (BCS) theory. The BCS Hamiltonian describes the interactions that lead to the formation of Cooper pairs. In the BCS theory, the superconducting state is characterized by the condensation of electrons into Cooper pairs. A Cooper pair is a bound state of two electrons with opposite momenta and opposite spins ($\vec{k}\uparrow -\vec{k}\downarrow$). The ground state wave function of a superconductor, in the simplest s-wave case, is a coherent superposition of states with different numbers of Cooper pairs. For unconventional superconductors, such as d-wave superconductors, the pairing symmetry is more complex. The pair wave function can have nodes or angular dependence. However, the spin part of the Cooper pair remains a spin singlet. The fundamental property that two electrons with opposite spins form a pair is preserved. When a Cooper pair is dissociated, for instance, in a mesoscopic structure or through transport experiments, the two constituent electrons can be spatially separated. If these electrons are sent to different locations (e.g., measurement devices or qubits), their entangled spin state can be used as a resource for quantum information protocols. The spin state of a single Cooper pair can be written as

$$|\Psi_{spin}\rangle = \tfrac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2) \tag{21}$$

This spin singlet state is a maximally entangled state, which is crucial for protocols like quantum teleportation. The superconducting environment provides a natural mechanism to generate and split such entangled pairs. However, Cooper pairs in a superconductor do not behave as maximally entangled states; instead, they are described by a Werner state.

For a system of interacting electrons in a metal, the Hamiltonian of s-and d-wave superconductors can be written as (throughout the paper, $k_B = \hbar = 1$ is considered) [12,16]

$$H = \sum_{ks} \varepsilon_k \, c^{\dagger}_{ks} c_{ks} + \sum_{k,k'} V_{k,k'} c^{\dagger}_{k\uparrow} \, c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} \tag{22}$$

where ε_k, $c^{\dagger}_{ks}$ and $c_{ks}$are the excitation energy with respect to chemical potential, the creation operator and the annihilation operator, respectively. For s-wave case, $V_{k,k'}$ is constant and independent of the momentum of each electron, meaning it has the same magnitude in all momentum directions. The interaction potential of the d-wave superconductor, $V_{k,k'}$, is given by [17]

$$V_{kk'}(\vec{v}_F, \vec{v}^{\,'}_F) = V_d \cos 2\,(\theta_k - \chi) \cos 2\,(\theta^{'}_k - \chi) \tag{23}$$

where $\vec{v}_F$, and χ are the Fermi velocity, and the angle between the crystallographic a-direction and the x-axis, respectively. Also, θ_k (and $\theta^{'}_k$) is the direction of $\vec{v}_F$ ($\vec{v}^{\,'}_F$) in the ab-plane. In addition, V_d is the dimensionless BCS constant of interaction λ_d given by $\lambda_d = \frac{V_d N(0)}{2}$, wherein $N(0)$is the density of states. Hamiltonian can be written in terms of gap energy, Δ_k, using mean field approximation as follows

$$\begin{aligned} H = \sum_{ks} \varepsilon_k \, c^{\dagger}_{ks} c_{ks} + \sum_{k,k'} \Delta_{k'} c^{\dagger}_{k\uparrow} \, c^{\dagger}_{-k\downarrow} \\ + \sum_{k,k'} \Delta^{*}{}_{k} c_{-k'\downarrow} c_{k'\uparrow} \end{aligned} \tag{24}$$

The low-temperature approximation used for the d-wave gap is valid in the regime $T \ll T_c$ (T_c is the critical temperature), where the magnitude of the superconducting gap is close to its zero-temperature value and thermal quasiparticle excitations are weak. The full gap function of a d-wave case with $d_{x^2 - y^2}$ symmetry is commonly written in the lattice form as $\Delta_k \equiv \Delta\left(\cos(k_x a) - \cos(k_y a)\right)/2$. For a nearly cylindrical Fermi surface, this expression reduces on the Fermi surface to the angular form $\Delta\left(\hat{k}^2_x - \hat{k}^2_y\right) = \Delta \cos(2\theta_k)$, where θ_k is the angle between the electron momentum and the gap axis and Δ is the magnitude of the gap. The gap changes sign under a π/2 rotation and vanishes along the nodal directions. The d-wave gap becomes to zero on the Fermi surface at 4 nodes. The order parameter with d-wave symmetry has 4 nodal points, $\vec{k}_i = \left(\frac{\pm k_F}{\sqrt{2}}, \frac{\pm k_F}{\sqrt{2}}\right)$, where the Fermi surface crosses the nodal directions $k_x = \pm k_y$. At these points, Δ_k becomes zero. At low temperature, the dominant low-energy excitations are located near these nodes. Therefore, close to a nodal direction, the gap can be expanded to first order in the angular deviation φ. This gives $\Delta_k = 2\Delta\varphi$ [18], where φ is measured from the nodal direction and Δ is the maximum gap amplitude. This linearized form is suitable for describing the low-energy behavior of the d-wave superconductor in the low-temperature limit.

To analyze the quantum correlations between electrons in a superconductor, it is useful to employ the Green-function and density matrix formalism. Green functions provide a compact way to describe both single-particle propagation and pairing correlations in many-body systems. In superconductors, there are two types of Green functions, i.e., the normal Green function and the anomalous Green function. The normal (single-particle) Green function is defined as [19]

$$\begin{aligned} &G_{\alpha\beta}(\vec{r}_1, t_1\,;\,\vec{r}_2, t_2) \\ &= -i\left\langle T\left[\psi_{\alpha H}(\vec{r}_1, t_1)\,\psi^{\dagger}_{\beta H}(\vec{r}_2, t_2)\right]\right\rangle \\ &\equiv \delta_{\alpha\beta} G(r_1, t_1\,;\,r_2, t_2) \end{aligned} \tag{25}$$

where $\psi_{\alpha H}(\vec{r}_1, t_1)$ and $\psi^{\dagger}_{\alpha H}(\vec{r}_2, t_2)$ are the electron annihilation and creation field operators for spin α in Heisenberg picture, and T denotes time ordering. This function describes the propagation of a single electron from point $\vec{r}_2$at time t$_2$ to point $\vec{r}_1$ at time t$_1$. In superconductors, however, the presence of Cooper pairing leads to additional correlations described by the anomalous Green function

$$\begin{aligned} &F_{\uparrow\downarrow}(\vec{r}_1, t_1\,;\,\vec{r}_2, t_2) = -i\langle T\,[\,\psi_{\uparrow H}(\vec{r}_1, t_1)\,\psi_{\downarrow H}(\vec{r}_2, t_2)\,]\rangle \\ &= I_{s_1=\uparrow, s2=\downarrow} F(\vec{r}_1, t_1\,;\,\vec{r}_2, t_2) \end{aligned} \tag{26}$$

where $I_{s_1=\uparrow, s2=\downarrow} = iY$. This anomalous function measures the amplitude for destroying two electrons with opposite spins at different space-time points. Physically, it represents the superconducting pair correlation.
Similarly, one may define

$$\begin{aligned} &F^{\dagger}_{\downarrow\uparrow}(\vec{r}_1, t_1\,;\,\vec{r}_2, t_2) \\ &= -i\left\langle T\left[\,\psi^{\dagger}_{\downarrow H}(\vec{r}_2, t_2)\,\psi^{\dagger}_{\uparrow H}(\vec{r}_1, t_1)\,\right]\right\rangle \end{aligned} \tag{27}$$

which describes the creation of a Cooper pair. These pair correlations are directly connected to the quantum entanglement between electrons. When a Cooper pair is split and its two electrons propagate to different spatial locations, the correlations encoded in the anomalous Green function translate into entanglement between the spins of the separated electrons. Therefore, Green functions provide the theoretical bridge between microscopic superconducting physics and quantum information quantities, such as the two-electron density matrix and entanglement measures. The quantum correlations between two electrons in a superconductor can be quantified by examining the two-particle reduced density matrix, which include both spatial and spin degrees of freedom. The two-electron density matrix is defined as [14,16]

$$\begin{aligned} &\rho^{(2)}(\vec{r}_1, \sigma_1; \vec{r}_2, \sigma_2; \vec{r}_{1'}, \sigma_{1'}; \vec{r}_{2'}, \sigma_{2'}) \\ &= \left(\frac{1}{2}\right)\langle \Psi^{\dagger}_{\sigma_{1'}}(\vec{r}_{1'}) \Psi^{\dagger}_{\sigma_{2'}}(\vec{r}_{2'}) \Psi_{\sigma_2}(\vec{r}_2) \Psi_{\sigma_1}(\vec{r}_1)\rangle \end{aligned} \tag{28}$$

within the BCS approximation, and focusing on the superconducting ground state, this quantity can be evaluated using Wick's theorem. Then

$$\rho^{(2)}_{s_1,s_2;s_1',s_2'}(r_1,r_2;r_1',r_2') = -\left(\frac{1}{2}\right)\Big[\delta_{s_1s_1'}\delta_{s_2s_2'}G(r_1 - r_1')G(r_2 - r_2') - \delta_{s_1s_2'}\delta_{s_2s_1'}G(r_1 - r_2')G(r_2 - r_1') - I_{s_1s_2}I_{s_1',s_2'}F(r_1 - r_2)F^*(r_1' - r_2')\Big] \tag{29}$$

Now applying $\vec{r}_1 = \vec{r}_1'$ and $\vec{r}_2 = \vec{r}_2'$, the two-electron spin-reduced density matrix can be expressed in the basis $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$as

$$\rho = \frac{1}{A}\begin{pmatrix} 1 - g(r)g(-r) & 0 & 0 & 0 \\ 0 & 1 & -g(r)g(-r) & 0 \\ 0 & -g(r)g(-r) & 1 & 0 \\ 0 & 0 & 0 & 1 - g(r)g(-r) \end{pmatrix} \tag{30}$$

where $A = 4 - 2g(r)g(-r)$, $g(r) = \frac{G(r)}{G(0)}$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the distance of two electrons in a Cooper pair. This matrix describes the mixed spin state of a pair of electrons extracted (or "split") from the superconductor. It should be clarified that the Werner state used here is not the full Cooper pair state. The full two-electron state contains both spin and spatial degrees of freedom. In the present work, only the reduced two-spin density matrix is used as the teleportation channel. The Werner form follows from the spin-singlet nature of Cooper pairing. In the superconductors considered in this work, the two electrons of a Cooper pair form a spin singlet. When the two electrons are taken at two different spatial points and the spatial degrees of freedom are not used as independent qubit variables, the reduced spin state becomes mixed. This mixed state contains a singlet contribution and a mixed background contribution, and it can be written in Werner form. This result holds within the assumptions of BCS mean-field theory, spin-singlet pairing, no magnetic impurities, no spin-dependent scattering, and no spin-orbit effects. Within this ideal model, the reduced two-spin density matrix has the Werner form. The spatial part of the problem enters through the superconducting Green functions, which determine the Werner parameter P. Therefore, the s-wave or d-wave gap symmetry does not change the basic Werner form of the reduced spin state, but it changes the value and spatial dependence of P. If effects such as magnetic disorder, spin-orbit coupling, or strong decoherence are present, the reduced spin state may deviate from the simple Werner form. In that case, the Werner-state description should be considered an approximation. Also, this is Werner state given by Eq. (5). Comparing Eqs. (5) and (30), for d-wave case, the parameter P is given by [16]

$$P_d = g(\vec{r})g(-\vec{r})/(2 - g(\vec{r})g(-\vec{r})) \tag{31}$$

For s-wave case, the parameter P

$$P_s = g(\vec{r})g(\vec{r})/(2 - g(\vec{r})g(\vec{r})) \tag{32}$$

Green's functions should be obtained numerically and then, from Eqs. (31) and (32), the parameter P can be obtained. The parameter $P$is the Werner parameter of the reduced two-spin density matrix of the two electrons extracted from a Cooper pair. In the present treatment, it satisfies $0 \le P \le 1$. Physically, P measures the weight of the spin-singlet component in the reduced two-electron spin density matrix. In an ideal Cooper pair, the two electron spins form a pure singlet state. However, when the two-electron density matrix is reduced to the spin variables at two spatial points, the result is generally a mixed state. The useful singlet correlation is then reduced by the spatial dependence of the superconducting Green functions. Therefore, P measures how strongly the two separated electrons still keep the spin-singlet correlation of the original Cooper pair. In this work, P is obtained from the superconducting Green functions and therefore depends on the electron separation, the superconducting gap, and the pairing symmetry. A larger value of P means a stronger useful spin correlation and a better teleportation channel. A smaller value of P means that the reduced two-spin state is more mixed and less useful for teleportation. Since the teleportation fidelity is determined by P, this parameter provides the connection between the microscopic superconducting correlations and the performance of the teleportation protocol.

The Werner-state description used here should not be understood as a complete neglect of the spatial or orbital degrees of freedom. The full two-electron state contains both spin and spatial parts. In the present teleportation protocol, however, the qubit is carried by the spin state of the two electrons. Therefore, the relevant channel is the reduced two-spin density matrix. The spatial and orbital parts enter the calculation through the superconducting Green's functions. These Green's functions determine how the two-electron correlations depend on the electron separation and on the superconducting gap symmetry. Their effect is therefore included in the Werner parameter P. Thus, the orbital degrees of freedom are not kept as independent qubit variables, but they are included through their effect on P. This point is especially important for the d-wave case. The angular structure of the d-wave gap changes the Green functions and leads to a different distance dependence of P. As a result, the orbital and gap-symmetry information still affects the teleportation fidelity, even though the final Werner state is written only in the reduced spin space.

This remarkable mapping from superconducting Green functions to the Werner mixed state allows one to apply well-developed quantum information measures, such as concurrence, entanglement of formation, and teleportation fidelity, directly in the context of condensed matter physics.

2.3. Fidelity of Teleportation

In quantum teleportation, the quality of the protocol is measured by the teleportation fidelity, defined as the overlap between the input state and the reconstructed output state, i.e.,

$$F = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle \tag{33}$$

Because the input state is unknown, one usually considers the average teleportation fidelity over all possible input states on the Bloch sphere. The average teleportation fidelity, F_{tele}, is mathematically defined as the average of the output fidelities over all possible pure states on the Bloch sphere [20-21]

$$F_{tele} = \int d\,\psi \langle \psi | \mathrm{E}(|\psi\rangle\langle\psi|) | \psi \rangle \tag{34}$$

where ⍰represents the quantum teleportation channel and $d\psi$ is the normalized measure on the Bloch sphere. To calculate the average teleportation fidelity of a quantum channel, it is not necessary to compute it for every individual state. The average teleportation fidelity of any shared state in a $d \times d$ dimensional space is directly related to its Fully Entangled Fraction, also known as the singlet fraction (f). The singlet fraction represents the overlap of the shared state (ρ_W) with a maximally entangled Bell state, i.e.,

$$f = \langle \psi^- | \rho_W | \psi^- \rangle \tag{35}$$

The general formula for average teleportation fidelity is

$$F_{tele} = \frac{d \cdot f + 1}{d + 1} \tag{36}$$

For qubits, the system dimension is $d = 2$. Thus, the teleportation fidelity formula becomes

$$F_{tele} = \frac{2f + 1}{3} \tag{37}$$

By substituting the definition of ρ_W into Eq. (32), one has

$$\begin{aligned} f &= \langle \psi^- | \left(p | \psi^- \rangle \langle \psi^- | + \tfrac{1-p}{4} I \otimes I \right) | \psi^- \rangle \\ &= p \langle \psi^- | \psi^- \rangle \langle \psi^- | \psi^- \rangle + \tfrac{1-p}{4} \langle \psi^- | I | \psi^- \rangle \end{aligned} \tag{38}$$

Since $|\psi^-\rangle$is normalized ($\langle \psi^- | \psi^- \rangle = 1$), the singlet fraction evaluates to

$$f = \frac{3p + 1}{4} \tag{39}$$

After substituting f from Eq. (39) into Eq. (37), the total average teleportation fidelity for the Werner channel becomes

$$F_{tele} = \frac{p + 1}{2} \tag{40}$$

This simple relation shows that the teleportation efficiency is determined entirely by the Werner parameter P. Using P given in Eqs. (31) and (32) for Werner state of s- and d-wave superconductors, the total average teleportation fidelity becomes

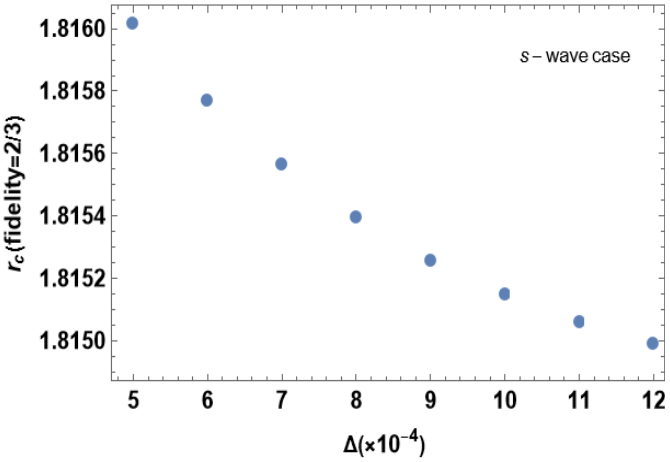

Fig. 1. The critical distance between the electrons of a Cooper pair, r_c, beyond which classical teleportation begins ($F_{tele} = \frac{2}{3}$), versus the superconducting gap size for s-wave case.

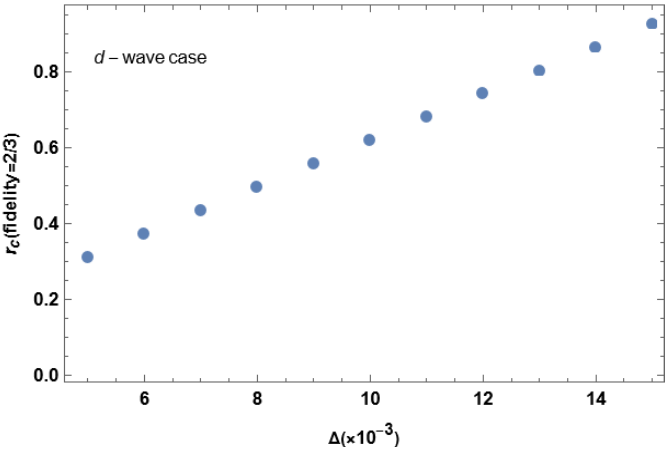

Fig. 2. The critical distance between the electrons of a Cooper pair, r_c, beyond which classical teleportation begins ($F_{tele} = \frac{2}{3}$), versus the superconducting gap size for d-wave case.

$$F_{tele}^{\text{s-wave}} = \frac{1}{2 - g^2(\vec{r})} \tag{41}$$

and

$$F_{tele}^{\text{d-wave}} = \frac{1}{2 - g(\vec{r}) g(-\vec{r})} \tag{42}$$

3. Results

An important criterion in quantum information theory is the classical teleportation limit, which represents the maximum fidelity achievable without using entanglement. This classical bound is

$$F_{classical} = \frac{2}{3} \tag{43}$$

Therefore, a quantum teleportation protocol is considered genuinely quantum only when

$$F_{tele} > \frac{2}{3} \tag{44}$$

The mapping to a Werner state should be distinguished from the condition for useful quantum teleportation. The Werner-state form is valid when the reduced two-spin density matrix is physical and the parameter P lies in its allowed range. However, teleportation above the classical limit requires a stronger condition. Using $F_{tele} = (1 + P)/2$ together with the classical bound $F_{classical} = 2/3$, one obtains $P > 1/3$. Thus, when $P \leq 1/3$, the state can still be written in Werner form, but it does not provide teleportation fidelity above the classical bound.

Now the classical limit and Eqs. (43) and (44) are used to plot Figures 1 and 2. These figures show the dependence of the critical distance between the electrons of a Cooper pair, r_c, beyond which classical teleportation begins, on the superconducting gap size for s-wave and d-wave cases, respectively. As can be seen, as the gap size increases within the considered range, this critical distance occurs at smaller values for the s-wave case and at larger values for the d-wave case.

In figure 3, for s-wave case, the quantum teleportation fidelity in terms of the distance of two electrons of a Cooper pair (in units of k_F) has been plotted. As seen, by increasing $k_F r$, the quantum teleportation fidelity, F_{tele}, decreased. When $k_F r$ becomes about 1.8, classical teleportation is possible. Also, in figure 4, for d-wave case, the quantum teleportation fidelity in terms of the distance of two electrons of a Cooper pair (in units of k_F) at different gap sizes has been plotted. It is worth noting that, by taking $\Delta = 0.001$ as the reference value, the other gap values shown in the figure can be interpreted as small fluctuations of the gap function around this value. The results therefore indicate that, fluctuations of the gap function can affect the behavior of the teleportation fidelity.

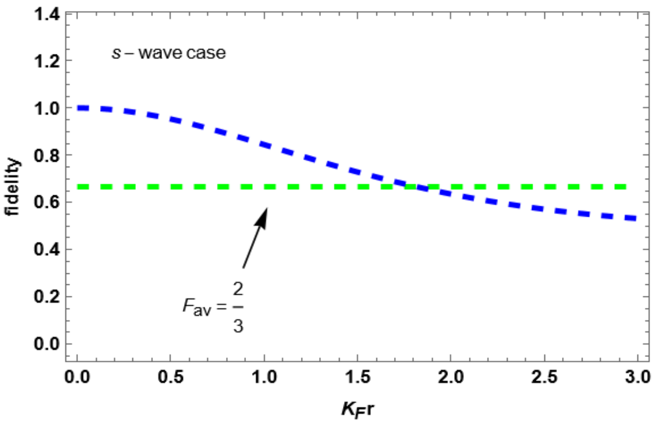

Fig. 3. The quantum teleportation fidelity in terms of the distance of two electrons of a Cooper pair (in units of k_F) for s-wave case.

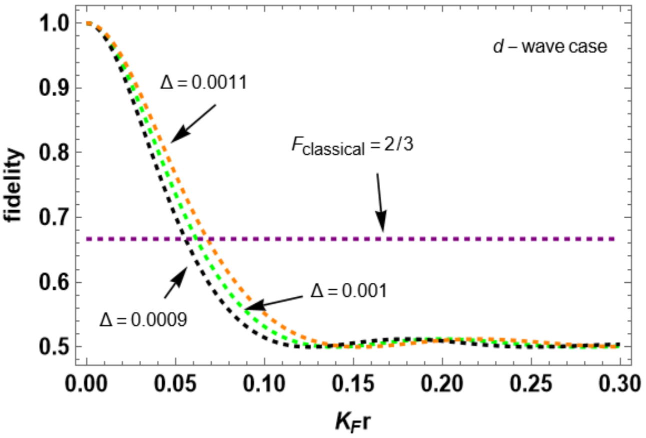

Fig. 4. For d-wave case, the quantum teleportation fidelity in terms of the distance of two electrons of a Cooper pair (in units of k_F) at different gaps~~;~~ (Δ (in units of Fermi energy, E_F~~;~~)) .

4. Conclusions

When two electrons originating from a superconducting Cooper pair remain sufficiently correlated (i.e., when the pair correlation function is large enough), the resulting entangled state can be used as a quantum channel for teleporting spin states. The efficiency of the teleportation protocol, therefore, depends strongly on how the pair correlation function decays with spatial separation and on the symmetry of the superconducting gap. Materials with longer coherence lengths and stronger pair correlations provide better conditions for maintaining entanglement over larger distances.

In this paper, we studied the teleportation of an unknown qubit using a Werner state that is linked to the Cooper-pair state in two types of superconductors: s-wave and d-wave. The teleported output is a single-qubit mixed state that depends on the parameter P. Therefore, it also depends on the electron–electron separation in a Cooper pair and on the superconducting gap.

Next, the relation between the teleportation fidelity and the superconducting Green's functions was derived for both the s-wave and d-wave cases, and the fidelity was then analyzed. For the s-wave superconductor, a critical distance between the two electrons was found such that at this distance and for larger distances, the teleportation becomes classical. This behavior is expected because the spatial superconducting correlations entering the reduced two-electron density matrix become weaker with increasing electron separation, which reduces the Werner parameter (P) and therefore lowers the teleportation fidelity. The presence of entanglement in the Cooper-pair electrons is the essential resource that allows quantum teleportation to outperform classical communication strategies. This result was expressed in terms of the gap, and it was shown that increasing the gap causes the onset of classical teleportation to occur at shorter distances. The different behavior of the s-wave and d-wave cases originates from the different structures of their superconducting gaps. Physically, in the s-wave case, the gap is isotropic and has no nodes on the Fermi surface. Increasing the s-wave gap reduces the characteristic coherence length of the Cooper pair. As a result, the useful superconducting correlations decay over a shorter distance, and the fidelity reaches the classical limit at a smaller electron separation. The same analysis was then carried out for the d-wave case as a function of the gap magnitude. In contrast to the s-wave case, it was found that as the gap magnitude increases, classical teleportation starts at larger distances. This different behavior is related to the anisotropic and sign-changing structure of the d-wave gap. In the d-wave case, the gap has nodal directions where it vanishes, and the pair correlation receives contributions from different momentum directions. These contributions can partly cancel each other at some distances and add more strongly at other distances. Therefore, the spatial dependence of the Green-function correlations, and consequently of the Werner parameter (P), is more complex than in the s-wave case. However, the characteristic distances in s-wave superconductors are, in general, much larger than the corresponding distances in d-wave superconductors.

Finally, the fidelity was studied as a function of distance. For the d-wave superconductor, three different gap values were considered. Since teleportation fidelity measures the similarity between the input state and the final state obtained using the Werner channel, the following behavior was observed. In the s-wave case, the fidelity decreases with increasing distance up to the point where classical teleportation begins. This behavior reflects the relatively smooth spatial decay of the superconducting correlations that determine the Werner parameter P. In the d-wave case, the behavior is more complex. The more complicated distance dependence comes from the angular structure of the d-wave pair correlation. As the electron separation changes, different momentum-direction contributions to the Green's functions can partly cancel or reinforce each other. Therefore, the Werner parameter P and the teleportation fidelity do not have to decrease smoothly with distance. For a fixed gap, the quantum teleportation fidelity first decreases with distance and reaches zero, but at a larger distance it becomes nonzero again. It then increases with distance, reaches a peak, and finally decreases to zero again at a new distance.

Outside the regime $T \ll T_c$, the full temperature dependence of the superconducting gap should be included. Thermal quasiparticles and scattering processes weaken the superconducting pair correlation. Consequently, the Werner parameter P decreases. Since the teleportation fidelity is determined by $F_{tele} = (1 + P) / 2$, finite-temperature effects are expected to reduce the teleportation fidelity and shift the quantum-to-classical crossover to shorter effective correlation distances. Therefore, the present results should be regarded as the ideal low-temperature limit of the superconducting teleportation channel. Disorder may also affect the channel. Magnetic disorder and spin-dependent scattering can directly disturb the singlet spin correlation. Nonmagnetic disorder is less harmful for an ideal s-wave superconductor, but it can still affect pair splitting. In d-wave superconductors, disorder can be more important because the gap is anisotropic and changes sign. Therefore, disorder can change the Green functions, reduce the effective P, and lower the fidelity.

This study of quantum teleportation using Cooper pairs, with the corresponding Werner state as the quantum channel, may help motivate the use of superconductors in quantum information applications.

Funding Statement

This research received no specific grant from any funding agency.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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